

$$\frac{+ b)^2 \cos 2(\alpha - u)]}{\cos 2u} \quad (30)$$

given by equation 33 [Hoek, 1965, 1].

$$\tau_t = -b/2a(\tan \alpha \pm \sec \alpha) \quad (33)$$

errors arise because some products of trigonometric functions of α removed by simplification of equation 30 are not neglected. The trigonometric functions take exact values.

A more elaborate analysis than Hoek's is required to determine the exact situation. It has not been attempted here. Instead, notice that physical considerations suggest that the maximum stress is at the crack tip when the crack major axis is parallel or perpendicular to the principal stress, and that equation 33 suggests that, in other positions, the maximum stress is at some distance from the crack tip.

The situation is more complex when the crack is inclined. Hoek [1965, p. 24] used the same assumptions as he made in the case of open cracks to show that on McClintock and Walsh's basis of the behavior of closed cracks,

$$\tau_t = P_1 \sin(\cos \alpha - m \sin \alpha) \quad (34)$$

where m is the coefficient of friction on the crack surface. The stress S_t is tensile for values of α greater than $m \sin \alpha$. Taking m to be equal to one, closed cracks inclined at more than 45° to P_1 will not, then, grow in uniaxial compression.

NEW THEORY OF BRITTLE CREEP

Now use this discussion of stress distribution around cracks and Charles's theory to explain brittle creep in uniaxial compression.

Assume that a subcritical crack in uniaxial compression extends in its own plane by stress σ due to the tensile stress near the crack tip and that when it reaches a critical length, it propagates in the manner described by Brace and Kohlstedt [1963].

This sequence may seem less plausible than the assumption that the crack grows along the path of maximum principal stress. However, the latter leads the crack to a stable configura-

tion without giving rise to any event that could cause the microseismic emission commonly observed in brittle creep. The principal contribution to creep strain comes from strains and displacements about propagating cracks. Once these cracks have propagated, they are stable or 'crack hardened.'

Sack [1946] has shown that results for stresses around flat cracks in two dimensions can be extended naturally to three dimensions, to flat cracks with a circular plan. These cracks have been termed 'penny-shaped' cracks. The maximum tensile stress on the crack margin lies in the plane of the minimum and maximum principal stresses and differs only by a constant from the value predicted by equation 30 for cracks in two dimensions.

Suppose there are $M(L, \alpha) dL$ cracks in the creep specimen with lengths at zero time between L_0 and $L_0 + dL$ at angles to the principal stress between α and $\alpha + d\alpha$. If each crack caused a strain increment v on propagating, the total strain de due to those cracks is $M(L, \alpha) v dL$. The time t_f for a crack length L_0 to grow to its critical length L_{cr} is given from equation 25 by

$$t_f = \exp(A/KT) L_{cr}^{n/2} \cdot [2/B(n-2)] L_0^{-1(n-2)/2} \quad (35)$$

$$t_f = E L_0^{-1(n-2)/2}$$

defining E .

Similarly, the time $(t_f - dt)$ for a crack of length $(L_0 + dL)$ to grow to L_{cr} is given by

$$(t_f - dt) = E(L_0 + dL)^{-1(n-2)/2} \quad (36)$$

Subtracting equation 36 from equation 35,

$$dt = E L_0^{-1(n-2)/2} \cdot [1 - (1 + dL/L_0)^{-1(n-2)/2}] \quad (37)$$

If dt and dL are small and n is large, equation 37 can be written

$$dt = [(n-2)/2] E L_0^{-n/2} dL \quad (38)$$

Then the strain rate at t_f due to the propagating cracks is $de/dt_f = M(L, \alpha) v dL/dt$,

$$de/dt_f = [2/E(n-2)] L_0^{n/2} \cdot M(L, \alpha) v \quad (39)$$

It would be reasonable to expect more short cracks than long ones. Thus $M(L, \alpha)$ is unlikely

to be independent of L_0 . Unfortunately, there is no direct way to determine the distribution of crack lengths.

Gilvarry [1961] suggested the basis of an indirect method. He considered the size distribution of the fragments in the single fracture of an infinitely extensive brittle-body due to the propagation of internal flaws. He divided internal flaws into three types, depending on the number of flaws against which they terminate. There are volume, facial, and edge flaws terminating against zero, one, or two flaws, respectively. Further classes were excluded because many of the fragments at fracture were four-sided. Gilvarry found

$$g = 1 - \exp[-(x/k) - (x/j)^2 - (x/i)^3]$$

where g is the volume (or weight) passing a mesh size x ; k, j, i are the average spacings of edge, facial, and volume flaws. If x is small, then this may be written,

$$g = 1 - \exp[-(x/k)]$$

and if x is very small,

$$g = (x/k) \quad (40)$$

so that fragments passing the smallest mesh size are controlled by edge flaws.

The weight dg in a size interval dx is given by differentiating equation 40,

$$dg/dx = 1/k \quad (41)$$

As $dg = Nk'L^3$ where N is the number of fragments in size interval, the number of fragments with average size L is given by

$$N = k''L^{-3} \quad (42)$$

Equation 42 has been confirmed experimentally by a number of workers [Gilvarry and Bergstrom, 1961]. In particular, Hamilton and Knight [1958] report the exponent of L to be about -2.75 for Pennant sandstone.

Single fracture has been defined by Gilvarry [1961] as 'fracture by an external stress system which is instantly and permanently removed when the first one or few Griffith cracks begin to propagate. Subsequent flaws are activated by stress waves produced by propagation of prior ones'. Assume, then, that the smallest fragments are bounded by flaws close to their original lengths so that the length distribution of the flaws can be written,